Thermo Quiz 2 | "The Junkers Jumo 004B" ®©™

hypotheses

- engine on the ground ($u_\infty = u_0 = 0$)
- around compressor and turbine, $T_t \approx T$, $p_t \approx p$ (kinetic energy neglected)
- potential energy effects neglected
- air modeled as a perfect gas (ideal gas + $\gamma = cst$) $\left(\frac{c_p}{c_v}\right)$
- ideal engine (100% efficient components)
- fuel neglected for accounting mass flows
- steady flow

Data

$T_{amb} = T_0 = 300K$

$P_{amb} = p_0 = 1 bar$

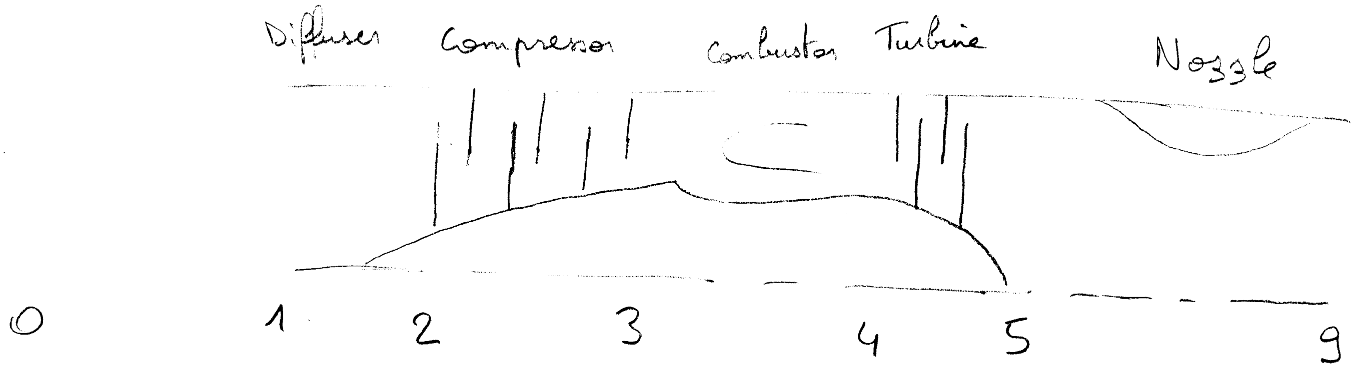
$\pi_{compressor} = 3$

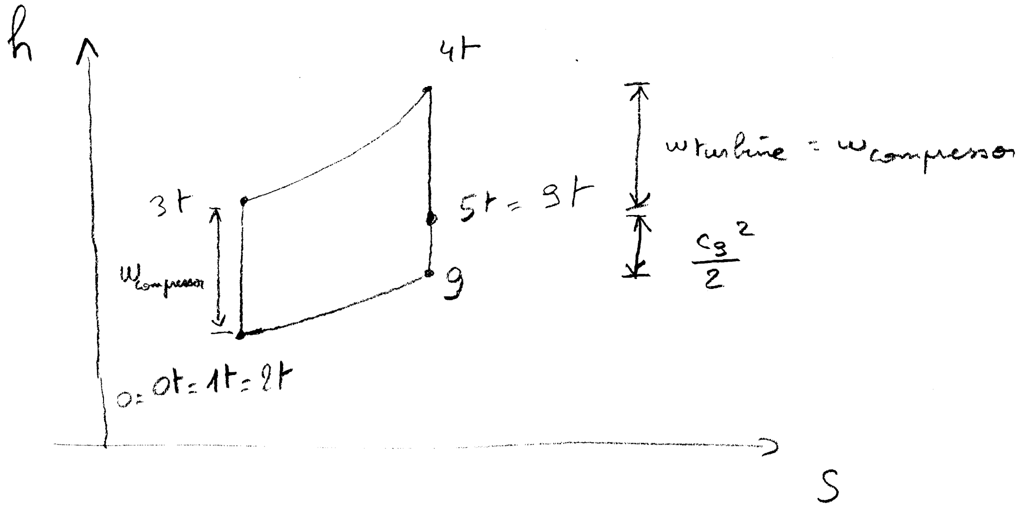
$\dot{m} = 20 kg/s$

$T_4 = 1050K$

$\gamma = 1.4$

$R = 287 J/kg \cdot K$





a) compressor exit (station 3)

Between stations 0 and 2 :

1st law, CV :

$$\cancel{\frac{dE_{cv}}{dt}} = \sum \cancel{\dot{Q}} + \sum \cancel{\dot{W}} + \dot{m}\left(h_{t_0} - h_{t_2}\right) + \dot{m}\, g\cancel{(z_0 - z_2)}$$

steady flow — no heat transfer — no work (no moving parts) — potential energy neglected

so that $h_{t_0} = h_{t_2}$, and for an ideal gas $\boxed{T_{t_0} = T_{t_2}}$

assuming ideal components (no losses), the process is adiabatic reversible , so that $\boxed{p_{t_0} = p_{t_2}}$

Since $M_0 = 0$, $\boxed{T_{t_2} = T_{t_0} = T_{amb} = 300K}$

$\boxed{p_{t_2} = p_{t_0} = p_{amb} = 1 bar}$

Through the compressor :

$$\Pi_{compressor} = 3 = \frac{p_{t_3}}{p_{t_2}} \quad \text{so} \quad \boxed{p_{t_3} = 3 \text{ bar}}$$

And for an ideal compressor (isentropic) :

$$\frac{T_{t_3}}{T_{t_2}} = \left(\frac{p_{t_3}}{p_{t_2}}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{so} \quad \boxed{T_{t_3} = 410.62\ K}$$

b) From 1st law, CV around the compressor :

$$\underbrace{\cancel{\frac{dE_{cv}}{dt}}}_{\substack{0 \\ \text{steady}}} = \underbrace{\sum \cancel{\dot{Q}}}_{\substack{\text{no} \\ \text{heat} \\ \text{transfer}}} + \underbrace{\sum \dot{W}}_{= -\dot{m}\, w_{compressor}} + \dot{m}\left(h_{t_3} - h_{t_2}\right)$$

since work is given to gas

so that for an ideal gas $\quad w_{compressor} = h_{t_3} - h_{t_2} = c_p\left(T_{t_3} - T_{t_2}\right)$

$$\boxed{w_{compressor} = \frac{\gamma R}{\gamma - 1}\left(T_{t_3} - T_{t_2}\right) = 111.12\ kJ/kg}$$

For an ideal shaft, $\boxed{w_{turbine} = w_{compressor} = 111.12\ kJ/kg}$

c) single stage turbine :

hypotheses :

$c_{0x} = c_{1x} = c_{2x}$

$u_1 = u_2$

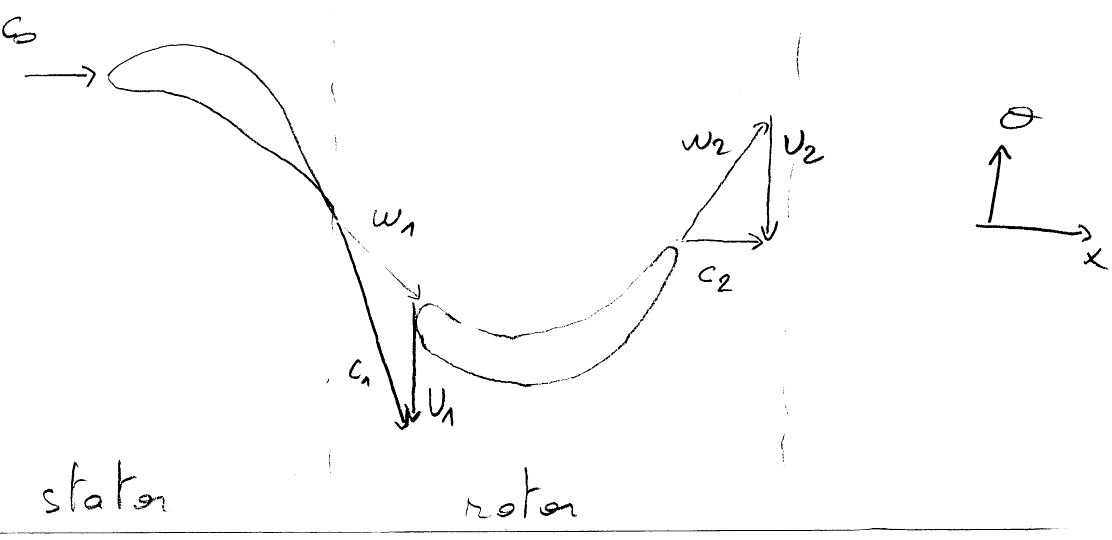
$c_2 = c_{2x}$ (no swirl) or $c_{2\theta} = 0$

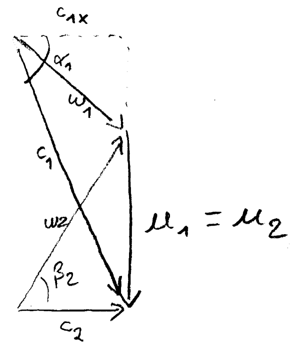
Data :

$u_1 = u_2 = (-)\,250\ m/s$

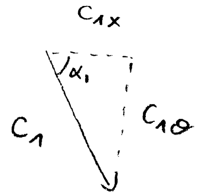
$\alpha_1 = 60°$

$\beta_2 = 50°$





d)



$$\sin \alpha_1 = \frac{c_{1\theta}}{c_1}$$

$$\Longrightarrow \boxed{c_1 = \frac{|c_{1\theta}|}{\sin \alpha_1}}$$

From Euler around the single stage turbine :

$$w_{turbine} = u\left(c_{1\theta} - \underbrace{c_{2\theta}}_{\text{no swirl}}\right) = u_1 c_{1\theta}$$

so that

$$\boxed{c_{1\theta} = \frac{w_{turbine}}{u_1} = (-)\ 444.47(7)\ m/s}$$

(negative θ direction)

and $\boxed{c_1 = \frac{|c_{1\theta}|}{\sin\alpha_1} = 513.23(77)\ m/s}$

Alternate solution

$$c_2 = c_{1x} = \frac{u_2}{\tan\beta_2} = \frac{u_2}{\tan 50^\circ}$$

$$\boxed{c_2 = c_{1x} = 209.77 m/s}$$

$$c_1 = \frac{c_{1x}}{\cos\alpha_1} = \frac{u_2}{\tan 50^\circ \cos 60^\circ}$$

$$\boxed{c_1 = 419.55\ m/s}$$

e) $$\boxed{c_2 = c_{2x} = c_{1x} = \left\{ \begin{array}{l} \sqrt{c_1^2 - c_{1\theta}^2} \\ \dfrac{|c_{1\theta}|}{\tan\alpha_1} \end{array} \right\} = 256.61(89)\ m/s}$$

Alternate: $\boxed{c_2 = u_2 / \tan 50^\circ = 209.77 m/s}$